5.4 Graphical Models

p. 310- 319

Rates of Change of an Investment- Investigation

Rates of Change for Investments

Sunita has $1000 that she saved from her summer job. She would like to invest the money. Her brother offers to borrow the entire sum at 10% per year simple interest so he can buy a new computer for college. Her bank offers a GIC that pays 8% per year, compounded annually. Which option should she choose?

1. **a)** Consider simple interest at 10% per year. Make a table showing the amount of the $1000 investment after each year for five years.
   b) What is the rate of change of the investment after the first year? the second year? the third year?
   c) Is the rate of change increasing, constant, or decreasing? What are suitable units for the rate of change?
   d) Is the relation modelled by a linear, a quadratic, or an exponential relation? Explain how you know.
   e) Graph the data with time on the horizontal axis and amount on the vertical axis.

Mar 24-11:12 AM

Simple Interest I= Prt

1000 (0.10)(1) = 100

1	100	1100
2	100	1200
3	100	1300
4	100	1400
5	100	1500

ii) 8% compounded annually annuity

$A = P(1+i)^n$

$A = 1000(1+0.08)^1$

$A = 1000(1.08)^1$

$A = 1080$

year 2 $A = P(1+i)^n$

$A = 1000(1+.08)^2$

1	1080	$A = 1000(1.08)^2$
2	1170	$A = 1000(1.08)^3$
3	1260	$A = 1000(1.17)$
4	1360	$A = 1170$
5	1470	

year 3, year 4, year 5

Apr 3-9:45 AM

2. **a)** Consider compound interest at 8% per year, compounded annually. Make a table showing the amount of the $1000 investment after each year for five years.
   b) What is the rate of change of the investment after the first year? the second year? the third year?
   c) Is the rate of change increasing, constant, or decreasing?
   d) Is the relation modelled by a linear, quadratic, or exponential relation? Explain how you know.
   e) Graph the data on the same set of axes used in part 1.
3. Use appropriate regression functions to determine the equation of a line or curve of best fit for each graph. Graph the lines or curves in the same window.
4. **Reflect** Compare the graphs. If necessary, adjust the window settings so that you can see all details. Where do the graphs intersect? Use the calculator to determine the coordinates of the intersection.
5. Based on your results, what advice would you give to Sunita?
6. **Reflect** Explain why your advice in step 5 depends on the length of time that Sunita plans to invest the money. Refer to rates of change in your answer.

Apr 3-7:29 AM

Example 1 p.311
Diagnosis of an Infection

Diagnosis of an Infection

An important part of medical diagnosis is determining the type of bacteria infecting a patient. Swabs are taken from suspected areas of infection and sent to a microbiology laboratory for testing. The bacteria are transferred to dishes containing nutrient-rich material and are allowed to grow. Suppose that dish A is infected with 100 bacteria and dish B is infected with 200 bacteria. Both types of bacteria grow at a rate of 20% per day.

a) The initial difference in the population of bacteria in the two dishes is 100 bacteria. Predict whether this difference will remain constant as the bacteria multiply. Justify your prediction.
b) Make a table showing the daily number of bacteria in each dish for one week. Determine the first differences for each population.
c) Compare the rates of change of the two populations. What are suitable units for the rates of change?
d) Draw a graph for each population on the same set of axes, with time on the horizontal axis and population of the vertical axis.
e) Was your prediction in part a) correct? Explain how you know, referring to the graph.

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5.4 Analyse Graphical Models • MHR 311

$y = ab^x$

Apr 3-7:30 AM

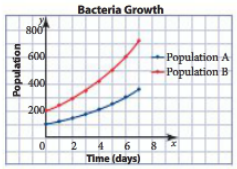
Solutions

b)

Time (days)	Population A	First Differences A
0	100	
1	120	20
2	144	24
3	173	29
4	207	34
5	249	42
6	299	50
7	358	59

Time (days)	Population B	First Differences B
0	200	
1	240	40
2	288	48
3	346	58
4	415	69
5	498	83
6	597	99
7	717	120

d)



Apr 3-7:31 AM

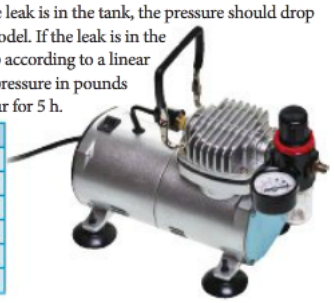
ii) Regression of Relationships

Example 2 using TI83 p. 313

Leaky Compressor

Dwight owns a construction business. He uses a compressor attached to a storage tank to maintain a supply of compressed air for power tools. He suspects a leak either in the tank itself or in the valve assemblies between the compressor and the tank. If the leak is in the tank, the pressure should drop according to an exponential model. If the leak is in the valve, the pressure should drop according to a linear model. Dwight measured the pressure in pounds per square inch (psi) every hour for 5 h.

Time (h)	Pressure (psi)
0	3000
1	2950
2	2900
3	2850
4	2800
5	2750



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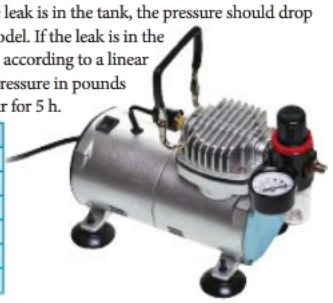

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a) Use the table to determine whether a linear or exponential model is more appropriate. What do you notice? Why does this happen?

b) Use technology to determine a linear and an exponential model for the data. Record the regression equations.

c) Display the graphs of both models. How long does it take before the graphs show a visible difference?

d) Dwight left the tank overnight. The next morning, after 14 h, the pressure was 2300 psi. Which model predicts this pressure most accurately?

e) Compare the rate of change of the linear model to that of the exponential model. Which model predicts that the pressure will decrease more slowly?

Apr 3-7:35 AM

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Apr 3-7:35 AM

Key Concepts

- *Rate of change on an exponential model depends on the initial conditions i.e. initial population & growth rate* and time
- *Prediction of future behaviour can help to identify rate of change and therefor the type of model being represented*

$y_1 \sim ax_1 + b$ R

$y_1 \sim ax_1^2 + bx_1 + c$ R^2

$y_1 \sim ab^{x_1}$ $\boxed{ab}$ R^2

Mar 21-12:30 PM

Hmk. p. 316 -319
q. 1-3, 5,6, 8 & 10 a)b)*

Mar 8-7:37 AM